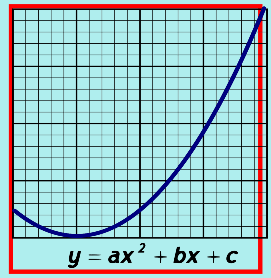
Math 25

Fall 2017

Lecture 13



Operations with Functions

① Addition $(f+g)(x) = f(x) + g(x)$

② Subtraction $(f-g)(x) = f(x) - g(x)$

③ Multiplication $(f \cdot g)(x) = f(x) \cdot g(x)$

④ Division $(f/g)(x) = \dfrac{f(x)}{g(x)}$; $g(x) \neq 0$

$f(x) = 4x - 3$, $g(x) = x + 4$

$(f+g)(x) = f(x) + g(x)$

$= 4x - 3 + x + 4 = \boxed{5x+1}$

$(f-g)(x) = f(x) - g(x)$

$= 4x - 3 - (x + 4) = 4x - 3 - x - 4$

$= \boxed{3x-7}$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$= (4x-3) \cdot (x+4) = 4x^2 + 16x - 3x - 12$

$= \boxed{4x^2 + 13x - 12}$

$(f/g)(x) = \dfrac{f(x)}{g(x)}$; $g(x) \neq 0$

$= \dfrac{4x-3}{x+4}$; $x + 4 \neq 0$

$x \neq -4$ Domain: All reals except -4.

$f(x) = x^2 + 1$, $g(x) = x - 2$

find $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$.

$(f+g)(x) = f(x) + g(x)$

$= x^2 + 1 + x - 2 = \boxed{x^2 + x - 1}$

$(f-g)(x) = f(x) - g(x)$

$= x^2+1 - (x-2)$

$= x^2+1-x+2 \quad = \boxed{x^2-x+3}$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$= (x^2+1)(x-2) = \boxed{x^3-2x^2+x-2}$

$(f/g)(x) = \dfrac{f(x)}{g(x)} \;;\; g(x) \neq 0$

$= \dfrac{x^2+1}{x-2} \;;\; x-2 \neq 0$ Domain

$x \neq 2$ All Reals except 2.

$\mathbb{R}$

Composition Operation $\circ$

$(f \circ g)(x)$ "f composition g of x"

$= f(g(x))$

ex: $f(x) = 3x+5$, $g(x) = 2x-3$

$(f \circ g)(x) = f(g(x)) = 3\,g(x)+5$

$= 3(2x-3)+5 = 6x-9+5$

$= \boxed{6x-4}$

$(g \circ f)(x) = g(f(x)) = 2f(x) - 3$

$= 2(3x+5) - 3 = 6x+10-3 = \boxed{6x+7}$

$f(x) = 2x - 4 \qquad g(x) = \frac{1}{2}x + 2$

Find $(f \circ g)(x) = f(g(x))$

$= 2\,g(x) - 4$

$= 2\left(\frac{1}{2}x + 2\right) - 4 = 2 \cdot \frac{1}{2}x + 2 \cdot 2 - 4$

$= \boxed{x}$

Find $(g \circ f)(x) = g(f(x))$

$= \frac{1}{2}\,f(x) + 2$

$= \frac{1}{2}(2x - 4) + 2 = \frac{1}{2} \cdot 2x - \frac{1}{2} \cdot 4 + 2$

$= \boxed{x}$

$f(x) = \sqrt{x+3} \quad , \quad g(x) = x^2 - 3$

$x \geq -3 \qquad x \geq 0$

Find $(f \circ g)(x)$ & $(g \circ f)(x)$

$(f \circ g)(x) = f(g(x)) = \sqrt{g(x) + 3}$

$= \sqrt{x^2 - 3 + 3} = \sqrt{x^2} = \boxed{x}$

$(g \circ f)(x) = g(f(x)) = (f(x))^2 - 3$

$= \left(\sqrt{x+3}\right)^2 - 3 = x + 3 - 3$

$= \boxed{x}$

If $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then $f(x)$ & $g(x)$ are inverse of each other.

$(f \circ f^{-1})(x) = x$ & $(f^{-1} \circ f)(x) = x$

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find $f^{-1}(x)$ of $\boxed{f(x) = \frac{x-2}{x}}$, $x \neq 0$

$y = \frac{x-2}{x}$ $x = \frac{y-2}{y}$ $xy = y - 2$

$xy - y = -2$ $y(x-1) = -2$

$y = \frac{-2}{x-1}$

	Domain	Range
$f(x)$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 1) \cup (1, \infty)$
$f^{-1}(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 0) \cup (0, \infty)$

$\boxed{f^{-1}(x) = \frac{-2}{x-1}}$; $x \neq 1$

$f(x) = \frac{x+4}{x-2}$, find $f^{-1}(x)$, make the table for domain & range using interval notation.

$f(x) = \frac{x+4}{x-2}$, $x - 2 \neq 0$, $x \neq 2$

$y = \frac{x+4}{x-2}$

$x = \frac{y+4}{y-2}$

$x(y-2) = y + 4$

$xy - 2x = y + 4$

$xy - y = 4 + 2x$

$y(x-1) = 2x + 4$

$y = \frac{2x+4}{x-1}$

$f^{-1}(x) = \frac{2x+4}{x-1}$

$x \neq 1$

	D	R
$f(x)$	$(-\infty, 2) \cup (2, \infty)$	$(-\infty, 1) \cup (1, \infty)$
$f^{-1}(x)$	$(-\infty, 1) \cup (1, \infty)$	$(-\infty, 2) \cup (2, \infty)$

Solve

1) $81^{2x-1} = 27^{3x}$

Notice: $81 = 3^4$, $27 = 3^3$

$(3^4)^{2x-1} = (3^3)^{3x}$

$3^{4(2x-1)} = 3^{3(3x)}$

$4(2x-1) = 3(3x)$

$8x - 4 = 9x$

$\boxed{-4 = x}$ $\{-4\}$

2) $3^{x-4} = 4$

Take log of both sides

$\log 3^{\boxed{x-4}} = \log 4$

$(x-4)\log 3 = \log 4$

$x - 4 = \dfrac{\log 4}{\log 3}$

Exact Ans

$\boxed{x = \dfrac{\log 4}{\log 3} + 4}$ $\boxed{x \approx 5.26}$

Solve

1) $\log_3(7-5x) = 2$

$3^2 = 7 - 5x$

$9 = 7 - 5x$

$2 = -5x$

$\boxed{\dfrac{-2}{5} = x}$

$\left\{\dfrac{-2}{5}\right\}$

2) $\log_4(3x+1) - \log_4(x-2) = 2$

$\log_4 \dfrac{3x+1}{x-2} = 2$

$4^2 = \dfrac{3x+1}{x-2}$

$16(x-2) = 3x + 1$

$16x - 32 = 3x + 1$

$13x = 33$ $\boxed{x = \dfrac{33}{13}}$ ✓

Find $f^{-1}(x)$ for $f(x) = \log_3(x-2) - 1$

$\to x-2>0$

$x>2$

Use table to express domain and range in interval notation.

$f(x) = \log_3(x-2) - 1$

$y = \log_3(x-2) - 1$

$x = \log_3(y-2) - 1$

$x + 1 = \log_3(y-2)$

$\log_3(y-2) = x+1$

$3^{x+1} = y-2$

$3^{x+1} + 2 = y$

$f^{-1}(x) = 3^{x+1} + 2$

	D	R
$f(x)$	$(2,\infty)$	$(-\infty,\infty)$
$f^{-1}(x)$	$(-\infty,\infty)$	$(2,\infty)$

Express as a single log:

$\log_2 7 - 2\log_2 x - \frac{1}{3}\log_2 y + \frac{1}{2}\log_2 z$

$= \log_2 7 - \log_2 x^2 - \log_2 y^{1/3} + \log_2 z^{1/2} = \boxed{\log_2 \frac{7\sqrt{z}}{x^2\sqrt[3]{y}}}$

Solve: $\underbrace{\log(3x+5) + \log(x-4)} = \log(2x^2-30)$

$\log_{10}(3x+5)(x-4) = \log_{10}(2x^2-30)$

$(3x+5)(x-4) = 2x^2-30$

$3x^2 - 12x + 5x - 20 = 2x^2 - 30$

$\to x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$\cancel{x=2}$ $\quad x=5$

Check $x=2$

$\log(2\cdot3+5) + \log(\cancel{2-4})$ $\to \log(-2)$ $\emptyset$ stop

Check $x=5$

$\log(3\cdot5+5) + \log(5-4) = \log(2\cdot5^2-30)$

$\log 20 + \log 1 = \log 20$ $\quad (\log 1 \to 0)$

$\log 20 = \log 20$ ✓

$\{5\}$

Consider 77, 68, 59, 50, ... $a_1 = 77$

$d = -9$

① find $a_n = a_1 + (n-1)d$

$= 77 + (n-1)(-9)$

$= 77 - 9n + 9 = \boxed{86 - 9n}$

② find $a_{80} = 86 - 9(80) = 86 - 720 = \boxed{-634}$

③ find $S_{80} = \frac{80}{2}(a_1 + a_{80})$

$= 40(77 - 634) = \boxed{-22\,280}$

Consider 7, 21, 63, 189, ... $a_1 = 7$

$r = 3$

find

1) $a_n = a_1 r^{n-1} = \boxed{7 \cdot 3^{n-1}}$

2) $a_{10} = 7 \cdot 3^{10-1}$

$= \boxed{}$

3) $S_{10} = \frac{a_1(1-r^n)}{1-r}$

$S_{10} = \frac{7(1-3^{10})}{1-3} = \boxed{}$

Find the Sum

Infinite

$20 + 8 + \frac{16}{5} + \frac{32}{25} + \cdots$

$a_1 = 20$

$r = \frac{8}{20} = \frac{2}{5}$

$S_\infty = \frac{a_1}{1-r} = \frac{20}{1-2/5} = \frac{20}{\frac{3}{5}} = \boxed{100/3}$

Find the term of $(3x - 4y)^{12}$ containing y^4.

$(A + B)^{12}$ with $A = 3x$, $B = -4y$

B^4

$\binom{12}{4} A^8 B^4$

the fifth term.

$495(3x)^8(-4y)^4 = \underbrace{495 \cdot (3)^8 (-4)^4}_{\text{compute that}} x^8 y^4$

$- - - - - - - \cdot x^8 y^4$

Use mathematical induction to prove

$2 + 8 + 14 + \cdots + (6n-4) = n(3n-1)$

Does it work for $n=1$?

$a_n = 6n - 4$

$n = 1 \quad a_1 = 6(1) - 4 = 2$ ✓

$1(3 \cdot 1 - 1)$

$1(3 - 1)$

$1 \cdot 2 = 2$ ✓

How about 2?

$a_2 = 6 \cdot 2 - 4 = 8$

$2 + 8 = 2(3 \cdot 2 - 1)$

$10 = 2 \cdot 5$ ✓

Let's assume it works for $n = k$, then show it works for $k+1$.

$$2 + 8 + 14 + \cdots + (6k-4) = k(3k-1)$$

Add the next term of the LHS to both sides.

$$\underbrace{2 + 8 + 14 + \cdots + (6k-4) + (6k-4+6)}_{\text{How many terms? } k+1 \text{ terms}} = k(3k-1) + (6k-4+6)$$

$= k(3k-1) + (6k+2)$

$= 3k^2 - k + 6k + 2$

$= 3k^2 + 5k + 2$

$= (k+1)(3k+2)$

$= (k+1)(3k+3-1)$

$= (k+1)(3(k+1)-1)$

Use mathematical induction to Prove

$$4 + 9 + 14 + \cdots + (5n-1) = \frac{n}{2}(5n+3)$$

$n=1$

$a_n = 5n-1$

$a_1 = 5(1)-1$

$= 4$ ✓

$\frac{1}{2}(5\cdot 1 + 3)$

$= \frac{1}{2}(5+3)$

$= \frac{1}{2}\cdot 8 = \boxed{4}$ ✓

$n=2$

$a_2 = 5(2)-1$

$= 9$

$4+9 = \frac{2}{2}(5\cdot 2+3)$

$13 = 13$ ✓

Assume it works for $n=k$, then check to see whether or not it holds true for $n=k+1$.

$$4 + 9 + 14 + \cdots + (5k-1) = \frac{k}{2}(5k+3)$$

Add the next term

$$4 + 9 + 14 + \cdots + (5k-1) + (5k-1+5) = \frac{k}{2}(5k+3) + (5k-1+5)$$

$$= \frac{k}{2}(5k+3) + \frac{2}{2}(5k+4) = \frac{k(5k+3)+2(5k+4)}{2}$$

$$= \frac{5k^2+3k+10k+8}{2} = \frac{5k^2+13k+8}{2} = \frac{(k+1)(5k+8)}{2}$$

$$= \frac{k+1}{2}(5k+8)$$

$$= \frac{k+1}{2}(5(k+1)+3)$$

Use mathematical induction to prove $7^n - 3$ is divisible by 2. → has a factor of 2.

$n=1 \rightarrow 7^1 - 3 = 7-3 = 4 = 2 \cdot 2$ ✓

$n=2 \rightarrow 7^2 - 3 = 49-3 = 46 = 23 \cdot 2$ ✓

Assume it works for $n=k$

$\Rightarrow 7^k - 3 = 2 \cdot F \qquad 7^k = 2 \cdot F + 3$

Now we want to show that $7^{k+1} - 3$ has a factor of 2.

$$7^{k+1} - 3 = 7 \cdot 7^k - 3 = 7(2 \cdot F + 3) - 3$$

$$= 7 \cdot 2F + 7 \cdot 3 - 3$$

$$= 14F + 18$$

$$= 2(7F + 9)$$

has a factor of 2.

Use M.I. to prove that

$5^n - 3$ has a factor of 2.

$n=1 \to 5^1 - 3 = 5 - 3 = 2 = 1 \cdot 2$

$n=2 \to 5^2 - 3 = 25 - 3 = 22 = 11 \cdot 2$

Assume it works for $n=k$

$5^k - 3 = F \cdot 2$

what about $5^{k+1} - 3$?

$5^{k+1} - 3 = 5 \cdot 5^k - 3 = 5(2F+3) - 3$

$= 5 \cdot 2F + 15 - 3$

$= 10F + 12 = (5F+6) \cdot 2$

Use M.I. to prove that $5n < 3^n$ for $n \geq 3$

Does it work for $n=3$? $5 \cdot 3 < 3^3$

$15 < 27$ ✓

what about $n=4$? $5 \cdot 4 < 3^4$

$20 < 81$ ✓

Assume it works for $n=k$

$5k < 3^k$

We need to show $5(k+1) < 3^{k+1}$

$\boxed{5(k+1)} = 5k + 5 < 5k + 10k$

$= 15k = 3 \cdot 5k < 3 \cdot 3^k$

$= \boxed{3^{k+1}}$

So $5(k+1) < 3^{k+1}$

Use M.I. to prove that
$3n < 2^n$ for $n \geq 4$.

Does it work for $n=4$? $3 \cdot 4 < 2^4$
$12 < 16$ ✓

$\underbrace{\text{Assume it works for } n=k}_{3k < 2^k}$

$\underbrace{\text{Now, we need to show it works for } n=k+1}_{3(k+1) < 2^{k+1}}$

$\boxed{3(k+1)} = 3k + 3 < 3k + 3k = 6k = 2(3k) < 2 \cdot 2^k = 2^{k+1}$

$3(k+1) < 2^{k+1}$

Next week
Exam III
Review your notes
Review Quizzes
Exam 1 & 2
} 10:00 – 12:00

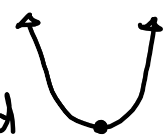
Lecture from 8:00 to 10:00
Look for new weekly Quiz
later tonight or Sunday morning

Quadratic Functions: $a \neq 0$

$f(x) = a(x-h)^2 + k$	$f(x) = ax^2 + bx + c$
$a>0$, $a<0$	$a>0$, $a<0$
Vertex (h,k)	$h = \frac{-b}{2a}$, $k = f(h)$
Axis of symmetry $x = h$	A.O.S. $x = h$
Y-Int $(0, \quad)$	Y-Int $(0, \quad)$
x-Int $(\quad, 0)$	x-Int $(\quad, 0)$

$f(x) = (x-3)^2 + 2$

$a=1$, $h=3$, $k=2$ opens upward

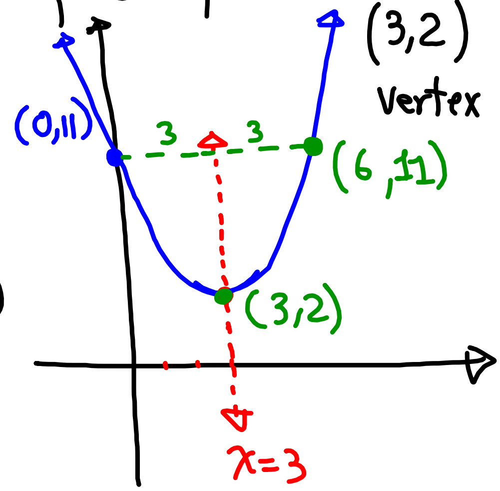


(3,2)

Vertex

A.O.S. $x=h$ $x=3$

Y-Int. $(0,11)$



x-Int (Always graph first)

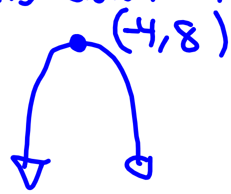
None

D: $(-\infty, \infty)$

R: $[2, \infty)$

Graph $f(x) = -\frac{1}{2}(x+4)^2 + 8$

$a = \frac{-1}{2}$, $h = -4$ $K = 8$ opens downward



A.O.S. $x = -4$

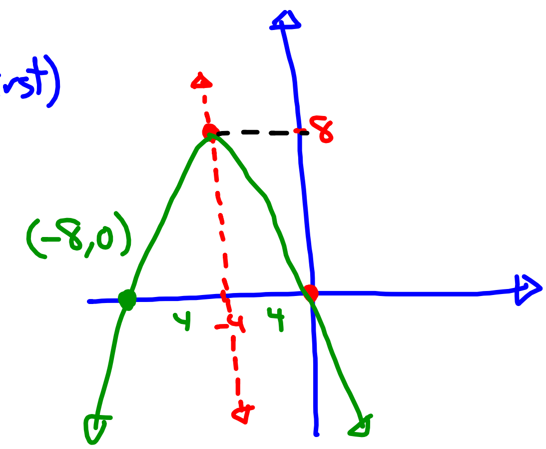
Y-Int $(0, 0)$

x-Int (Always graph first)

$(0,0)$, $(-8,0)$

D: $(-\infty, \infty)$

R: $(-\infty, 8]$



find x-Ints $y = 0 \rightarrow f(x) = 0$

Solve

$-\frac{1}{2}(x+4)^2 + 8 = 0$

$-\frac{1}{2}(x+4)^2 = -8$

multiply by -2

$(x+4)^2 = 16$

Square-root method

$x + 4 = \pm\sqrt{16}$

$x + 4 = \pm 4$

$x = -4 \pm 4$

$x = -4-4$ $x = -4+4$

$\boxed{x = -8}$ $\boxed{x = 0}$

$(-8,0)$

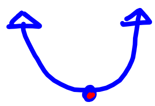
$(0,0)$

$f(x) = x^2 - 4x + 4$

$a = 1$ $\quad b = -4$ $\quad c = 4$

$h = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

$a > 0$



$(2,0)$

$k = f(h) = f(2) = 2^2 - 4(2) + 4 = 0$

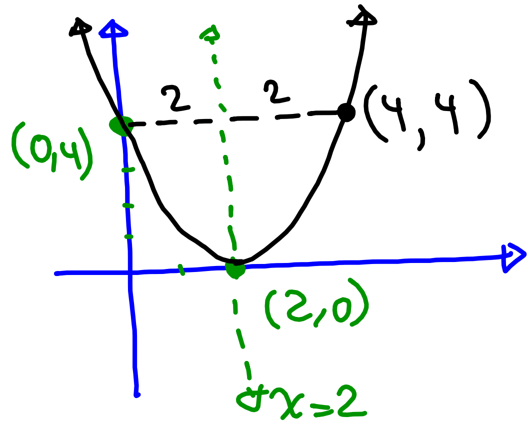
A.O.S. $\quad x = h \quad x = 2$

Y-Int $\quad (0, 4)$

x-Int $\quad (2,0)$

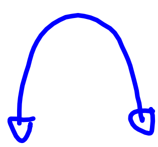
D: $(-\infty, \infty)$

R: $[0, \infty)$



Graph $\quad f(x) = -x^2 + 2x - 4$

$a = -1$ $\quad b = 2$ $\quad c = -4$



$h = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

$k = f(1) = -1 + 2 - 4 = -3$ $\quad \Rightarrow$ vertex $(1, -3)$

A.O.S. $\quad x = 1$

Y-Int $\quad (0, -4)$

x-Ints $\quad$ None

D: $(-\infty, \infty)$

R: $(-\infty, -3]$